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How do student and classroom characteristics affect attitude toward mathematics? A multivariate multilevel analysis

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ABSTRACT

This study investigated the effects of student and classroom characteristics on math self-confidence, perceived usefulness, and enjoyment of mathematics as multiple outcomes. A sample of 7th-grade students from 78 classes of 49 schools was studied. The data were collected using, among other instruments, an attitude questionnaire. The results of the multivariate multilevel analysis showed that the variance of the 3 indicators was situated mostly at the student level, and that the indicators correlated strongly at the class level. Higher prior mathematics achievement and positive parental beliefs and attitudes were significant predictors of higher scores across the 3 indicators. Each of the baseline indicators was significantly associated with its corresponding final indicator. At the classroom level, classroom assessment was significantly associated with less endorsement of all 3 indicators, higher levels of classroom modeling with greater endorsement of perceived usefulness of mathematics, and classroom questioning with greater enjoyment of mathematics.

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Attitude; enjoyment and usefulness of mathematics; mathematics self-confidence; multivariate multilevel modeling; Uganda

Introduction

Research in social sciences and educational psychology acknowledges the multifaceted structure of attitude toward mathematics (ATM; Di Martino & Zan, 2010; Hart, 1989; Kadijevich, 2008), although some researchers analyze ATM as a single outcome (e.g., Eshun, 2004). Other researchers explicitly make a distinction between multiple dimensions. For instance, a study conducted by Vandecandelaere, Speybroeck, Vanlaar, De Fraine, and Van Damme (2012) investigated the relation between the learning environment and three aspects of ATM, which they called: mathematics self-concept, enjoyment of mathematics, and perceived value of mathematics. They carried out a series of univariate multilevel analyses of each of the three indicators.

In this paper, we overcome these limitations of previous research by considering the three attitudinal indicators of mathematics self-confidence, perceived usefulness of mathematics, and enjoyment of mathematics as multiple outcomes. These three dimensions can be clearly separated (Vandecandelaere et al., 2012). *Mathematics self-confidence* concerns the students' confidence in their ability to learn and perform well on mathematics tasks. *Perceived usefulness of mathematics* refers to the students' beliefs about the importance of mathematics in every day and later life.

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Enjoyment of mathematics contains the extent to which the student enjoys mathematics lessons and the subject matter itself. Eshun (2004) observed that students are likely to achieve better in a subject that they enjoy, have confidence in, or find useful.

This study uses data from students clustered within classes, and thus the data have a multilevel structure. In recognition of the hierarchical structure of the data, the above considerations have prompted us to adopt a multivariate multilevel (MVML) analysis approach, analyzing the three outcomes simultaneously. The MVML allows us (a) to decompose the residual variances and covariances among the three indicators into student-level and class-level components (Grilli, Pennoni, Rampichini, & Romeo, 2014; Snijders & Bosker, 2012); (b) to test for significant correlations between a set of predictors and a set of responses (three indicators) at different levels of analysis while accounting for the correlations among the responses; (c) to compare the effect of each predictor on the three response variables (Snijders & Bosker, 2012; Tabachnick & Fidell, 2007). The aim of this study is to analyze the factors that affect these three attitudinal indicators among 1st-year secondary school students in Central Uganda, paying particular attention to gender differences in students' ATM because some studies have reported that girls have lower self-confidence in mathematics than boys (e.g., Asante, 2012; Eshun, 2004).

The rest of the paper is structured as follows. We review the related literature and formulate the research questions. We describe the explanatory variables at the student and classroom level. We then specify the multivariate multilevel model to be estimated. Results of the parameter estimates for both the fixed and random effects are presented. A discussion and concluding remarks are offered in the final section.

Setting

First, we give some information on the local context of our research. Examining ATM remains a critical concern in most developing countries, like Uganda. Studies on ATM have mostly been conducted in developed countries while very few studies have been undertaken in Africa (Chamdimba, 2008; Opolot-Okurut, 2005; Yara, 2009).

Two of the main objectives of teaching mathematics in Uganda include: developing a positive ATM and gaining high mathematics achievement (MA; Opolot-Okurut, 2005). These two objectives are considered to be mutually reinforcing. Despite its usefulness and importance, mathematics is perceived by an important part of students as difficult, boring, impractical, abstract, and so forth (Ignacio, Blanco Nieto, & Guerrero Barona, 2006). Such negative attitudes partially explain the students' persistent low-performance in mathematics (Kiwanuka, Van Damme, Van den Noortgate, Anumendem, & Namusisi, 2015). For example, the Uganda National Examinations Board (2013) reported that out of 286,166 candidates who sat for 2013 Ordinary Level examinations, 66.3% passed mathematics, whereas 74.6% passed English language. Therefore, it is important to conduct this study so as to examine the predictors of the three attitudinal indicators, and thus provide evidence to inform the current educational system in Uganda.

Literature review

Attitudes toward mathematics as a construct

In mathematics education, many studies have attempted to define and redefine attitude as a construct (Di Martino & Zan, 2010; Haladyna, Shaughnessy, & Shaughnessy, 1983; Hannula, 2002; Hart, 1989). The multidimensional nature of ATM gives rise to its multiple definitions. ATM has been defined as a positive or negative emotional disposition towards mathematics (Haladyna et al., 1983; Zan & Di Martino, 2007). Eshun (2004) defines ATM as "a disposition towards an aspect of mathematics that has been acquired by an individual through his or her beliefs and experiences but which could be changed" (p. 2). A bi-dimensional definition describes ATM as the pattern of beliefs and emotions associated with mathematics (Daskalogianni & Simpson, 2000; Fennema & Sherman, 1976). Empirical studies consider attitude as a multidimensional construct. Hart (1989) defines ATM in terms of three dimensions: an affective, a cognitive, and a conative (behavioral) dimension. For Hart, students' ATM is a combination of the emotions that they associate with mathematics, their beliefs towards mathematics, which could be either positive or negative, and how they behave towards mathematics.

Di Martino and Zan (2010) assume a cause-and-effect relationship among the three components in a chain: beliefs affect emotions, which in turn affect behavior. Kadijevich (2008) distinguishes the three dimensions as follows:

> (1) self-confidence denotes perceived ease, or difficulty, of learning mathematics, (2) liking (enjoying) mathematics stands for student's affective, emotional and behavioral reactions concerning liking, or disliking mathematics,(3) usefulness of math denotes students' beliefs concerning the contribution of mathematics to his/her educational and vocational performance. (p. 330)

Students' confidence in mathematics, mathematics anxiety, motivation in mathematics, and usefulness and enjoyment of mathematics have been documented as important indicators of students' ATM (Fennema & Sherman, 1976; Tapia & Marsh, 2004). According to Mohammadpour (2012), students' MA can be affected by whether the students find mathematics enjoyable, valuable, and important for success in school and for future career aspirations. Midgley, Feldlaufer, and Eccles (1989) observed that students' perceptions of the usefulness and importance of math are associated with their math-related performance. For this study, we define ATM as an aggregated measure of mathematics self-confidence, perceived usefulness, and enjoyment of mathematics. Mathematics self-confidence and perceived usefulness of mathematics reflect the cognitive dimension of attitude. Enjoyment of mathematics reflects the affective dimension of attitude.

Influences on ATM

Student level

Factors such as parental educational level and parental occupation (Köğce, Yildiz, Aydin, & Altındağ, 2009; Mohamed & Waheed, 2011) influence students' ATM. Literature focusing on parents' ATM suggests that their attitude can affect students' attitudes toward the subject (Beswick, 2006; Schoenfeld, 1985). Not surprisingly, literature suggests that negative parents' ATM can play a role in the negative attitudes among their children (Goodykoontz, 2009; Uusimaki & Nason, 2004). Cai, Moyer, and Wang (1997) reported that students with the most supportive parents show a more positive ATM than students with the least supportive parents.

Many studies have found no significant difference in ATM between male and female students (Köğce et al., 2009; Mohd, Mahmood, & Ismail, 2011). In their study of secondary school students in Pakistan, Farooq and Shah (2008) found no significant difference in confidence in mathematics between male and female students. However, other studies have evidence of significant differences in ATM, with girls showing a more negative attitude toward the subject than boys (Frost, Hyde, & Fennema, 1994). Moreover, these attitudes tend to become increasingly negative as students move from elementary to secondary school (McLeod, 1992). Kaiser-Messmer (1993) observed significant gender differences in favor of boys in interest/enjoyment of mathematics. However, Sanchez, Zimmerman, and Ye (2004) reported that American girls perceived mathematics as more important than boys. Some studies give evidence that compared to boys, girls lack confidence in doing mathematics and view mathematics as a male domain and are anxious about mathematics (Asante, 2012; Hyde, Fennema, Ryan, Frost, & Hopp, 1990).

Charles, Harr, Cech, and Hendley (2014) found that older eighth graders are more likely to have a positive ATM than their younger counterparts. Similarly, Chaman and Callingham (2013) found significant differences between students in the same class for ATM in favour of older students.

Studies that have been conducted to determine the relationship between students' ATM and MA have yielded contradictory results (Mensah, Okyere, & Kuranchie, 2013). Some studies have shown a strong and significant relationship between these two variables (Abosalem, 2015; Bakar et al., 2010; Else-Quest, Hyde, & Linn, 2010; Mahanta & Islam, 2005; Mensah et al., 2013; Michelli, 2013; Mohamed & Waheed, 2011; Mohd et al., 2011; Sarwar, Bashir, & Alam, 2010; Tapia & Marsh, 2004; Van den Broeck, Opdenakker, & Van Damme, 2005; Winheller, Hattie, & Brown, 2013). Many studies demonstrate that the three aspects of ATM have emerged as salient predictors of MA (Ampad, 2009; Klassen & Chiu, 2010; Ma & Xu, 2004; Mullis, Martin, Foy, & Arora, 2012; Tapia & Marsh, 2004; Van Damme, Opdenakker, & Van den Broeck, 2004; Vandecandelaere et al., 2012; Williams & Williams, 2010). Literature has also evidence of a reciprocal causal relationship between attitude toward mathematics and mathematics achievement (Ma & Kishor, 1997; Ma & Xu, 2004; Minato & Kamada, 1996). A recent study indicates that the effect of MA on ATM might be stronger than the effect of ATM on MA (Garon-Carrier et al., 2016). To evaluate the magnitude of the ATM–MA relationship, Ma and Kishor (1997) conducted a meta-analysis on 113 studies. They found that the overall mean effect was positive, with a weaker association in primary schools and a stronger in secondary schools. That the correlation between ATM and MA has been found to be low has led some researchers to conclude that it could not be considered to be of practical significance (Ma, 1997).

Classroom level

Inspired by educational effectiveness research (EER), Creemers and Kyriakides (2008) proposed a dynamic model to study educational settings and outcomes. Next to factors at the student, school, and system levels, this model refers to eight classroom-level factors which describe teachers' instructional role and are associated with student outcomes: *orientation, structuring, questioning, modeling, application, management of time, teacher role in making classroom a learning environment, and assessment*. These eight factors are usually measured through student questionnaires. Kyriakides et al. (2014) used student questionnaires to measure the teaching of mathematics and science in six European countries (i.e., Belgium/Flanders, Cyprus, Germany, Greece, Ireland, and Slovenia) and found that student ratings are reliable and valid.

Many studies have identified classroom-level characteristics that were found to influence students' ATM. These include teaching techniques (Elliott, Oty, McArthur, & Clark, 2001; Whitin, 2007) and classroom learning environment (Fisher & Richards, 1998). Goodykoontz (2009) reported that the amount and type of assessments were cited by students as affecting their enjoyment and liking of mathematics. Miska (2004) cited classroom modeling as one of the relevant teaching methods. Teaching style and the teacher's behavior are reported to affect students' ATM (Midgley et al., 1989; Wanzer & McCroskey, 1998). Brualdi-Timmins (1998) found that classroom questioning is one of the most popular teaching styles, which influences academic achievement and attitude. Higgins (1997) observed that, compared with the students who had received traditional mathematics instruction, the students who had received problem-solving instruction displayed more positive attitudes about the usefulness of math.

Sanchez et al. (2004) found a moderate relationship between friends' or classmates' attitudes and perceived importance of, and interest in studying mathematics. Students' ATM is affected by teachers' beliefs and attitudes toward mathematics (Beswick, 2007; Wilkins & Brand, 2004). The study of Mensah et al. (2013) reported that the more positive the attitude of mathematics teachers toward the subject, the more positive the students' attitude toward the study of the subject.

Considering the mentioned research literature, we focused on four of the eight classroom-level factors of the dynamic model, that is, learning environment, assessment, questioning, and modeling. Additionally, we considered also teacher beliefs and influence of peers.

Research questions

The current study was guided by the following research questions:

(1) What is the association among mathematics self-confidence, perceived usefulness, and enjoyment of mathematics at student and classroom level?
(2) To what extent do student intake characteristics (socioeconomic status, parents' beliefs, gender, age, prior mathematics achievement, prior self-confidence, prior usefulness perception, prior enjoyment) explain differences in (a) mathematics self-confidence, (b) perceived usefulness of mathematics, and (c) enjoyment of mathematics?
(3) To what extent do class processes (classroom learning environment, classroom assessment, classroom questioning, classroom modeling, teacher beliefs, and peer influence) and class composition variables (proportion of girls in the class, class mean of student intake characteristics) explain differences in (a) mathematics self-confidence, (b) usefulness of mathematics, and (c) enjoyment of mathematics, after controlling for student intake-characteristics?

Methodology

Study sample

Our study used a multistage sampling design. At the first stage, four districts from the Central region of Uganda were purposefully selected. These were: Kampala and Wakiso, two urban and populated regions with people from different parts of the country, and Mpigi and Mukono, two rather rural but reachable regions. At the second stage, 60 schools were randomly selected from a total of 376 schools in the four districts with 25 semi-rural and 35 urban schools. These schools followed the same curriculum, but with a variety of learning climate and teacher practices. At the third stage, classes were selected. Three or four classes were randomly selected from schools with four or more Grade 7 classes, two from schools with three classes, and all classes in schools with only one or two classes. Forty-nine (82%) of the sampled schools participated in the study. Within these schools, all 78 selected classes and all students from these classes participated. Table 1 shows the number of schools and sampled classes in target and participating sample. Table 2 shows some

Table 1. Number of schools and sampled classes in target and participating sample.

	Target sample		Participating sample	
Total no. of 7th-grade classes per school	No. of schools	No. of classes per school	No. of schools	Total no. of classes
5	4	4	2	8
4	8	3	6	18
3	8	2	5	10
2	7	2	6	12
1	33	1	30	30
Total	60		49	78

Table 2. Categories of participating schools.

		No. of schools
By type	Government	12
	Private	37
By location	Urban	28
	Semi-urban	21
By gender	All boys	2
	All girls	2
	Co-educational	45

characteristics of the participating schools. The numbers of semi-rural schools were: 12 from Mpigi, 9 from Mukono, and 4 from Wakiso; and the numbers of urban schools were: 18 from Kampala and 17 from Wakiso.

There were 4,819 first-year secondary school students (Grade 7; about 14–15 years old) who participated at one or two measurement occasions. There were 2,170 (45%) boys and 2,649 (55%) girls.

Instruments

The instruments used for this study were a student questionnaire administered at the beginning and at the end of School year 2012, and a math test administered at the beginning of the same year. The mathematics test was based on a selection of the freely available items of the Trends in International Mathematics and Science Study (TIMSS) 2011. A curriculum-relevant selection was made, which ended up with 23 items in total of which 10 multiple-choice questions and 13 items derived from 8 open questions. The test was designed to measure numeration, algebra, fractions, geometry, and word problems. A pilot study was administered to students from two schools, and teachers were asked to evaluate the appropriateness of the items. The student questionnaire was an instrument designed to collect demographic information based on the Programme for International Student Assessment (PISA) 2009 student questionnaire, perceptions of classroom teaching of mathematics based on the student questionnaire developed and used by Kyriakides and colleagues (2014) to assess the teaching of mathematics and science, and perceptions of teacher attitudes and beliefs, peer influence, parental beliefs, and self-reported attitudinal factors, modified from the Fennema-Sherman Mathematics Attitude Scales (FSMAS; Fennema & Sherman, 1976) and the Attitudes Toward Mathematics Inventory (ATMI; Tapia & Marsh, 2004). The last five scales consisted of items to be judged on a 5-point Likert scale (1 = *strongly disagree* to 5 = *strongly agree*). Additional details of the variables and scales are provided below and in Appendix 1.

Data collection

At the beginning of the academic year, the student questionnaire was administered along with the math test. The researcher and/or his assistants administered the test and questionnaires to the students with the help of mathematics teacher(s) during mathematics class time. The students were assured of confidentiality and informed that the data collected in the study would be only for research purposes.

Measures

Dependent variables

Mathematics self-confidence (CONF, 10 items, Cronbach's α observed in our study = .70), perceived usefulness (USE, 6 items, α = .73), and enjoyment of math (ENJOY, 6 items, α = .60) are the dependent variables in our study. The exploratory analysis shows that the three outcomes are positively correlated over students and classes, with observed correlations between .43 and .54 (see Appendix 2).

Descriptive statistics for the three indicators of ATM of boys and girls at two measurement occasions are provided in Table 3. The scores were scaled to a mean of 50 and deviation of 10 using T-transformation, and they range from 0 to 100 (J.R. Graham, Naglieri, & Weiner, 2003). Hence, raw scores are transformed to standard scores for easy comparisons across self-report inventory scales.

Table 3 shows that there was a decline in mathematics self-confidence and enjoyment of mathematics for both boys and girls from Time 1 to Time 2, and a slight increase in usefulness perception of mathematics. The results from the two-sample t test showed that each of the three indicators (math

Table 3. Descriptive statistics of the attitudinal indicators by gender.

	Time 1				Time 2			
	Boys N = 2146		Girls N = 2622		Boys N = 1887		Girls N = 2357	
Indicator	M	SD	M	SD	M	SD	M	SD
CONF	67.67	18.96	67.91	18.93	61.21	17.43	60.40	16.92
USE	73.47	18.46	76.60	17.48	75.44	18.70	77.27	18.46
ENJOY	68.12	19.81	68.79	19.93	63.91	19.19	62.21	20.00

CONF = Math self-confidence; USE = usefulness of Math; ENJOY = enjoyment of Math.

self-confidence, perceived usefulness, and enjoyment of mathematics) had statistically significant differences between their prior and later means, $t(4192) = 21.39$, $p < .001$, $t(4192) = -2.24$, $p = .025$, and $t(4192) = 15.57$, $p < .001$, respectively.

Explanatory variables at the student level

We used a principal component analysis to construct a single socioeconomic-status variable (SES, α= .76) as a weighted composite index of five variables: educational level of father and mother, occupational status of father and mother, and home possessions (phone, television, computer, and car). Both the educational level of the mother and that of the father show the highest factor loadings on the created SES variable. Gender was a dummy variable coded 0 for boys and 1 for girls. Prior mathematics achievement (PMA) was measured with a mathematics test taken at the beginning of School year 2012 (23 items, α = .70). Each item of the test was scored 1 if correct or 0 if wrong. Based on the scores of a sample of 720 students, the item difficulty and discrimination parameters from the two-parameter logistics (2 PL) item response theory (IRT) model were estimated, using the maximum likelihood procedure implemented in the latent trait model (ltm) package from R. The raw scores were converted into maximum likelihood ability estimates. The abilities were transformed into a scale with a mean of 50 and a standard deviation of 10 using the T-transformation.

For the dependent and independent scales, some items were removed because of low-total correlation so as to increase the Cronbach's alphas of the scales. A confirmatory factor analysis (CFA) was performed to determine the structure of the scales at each time point. The retained factor loadings were significant (p < .05). The principal component analysis and CFA were used to establish the reliability and validity of the scales. Hence, other scales were prior self-confidence (PRCONF, 10 items, α = .71), prior perceived usefulness (PRUSE, 6 items, α = .61), prior enjoyment (PRENJOY, 6 items, α = .57), and parental beliefs and attitudes (PABELF, 10 items, α = .79). The scale of PABELF measures students' perceptions of their parents' beliefs, interest, encouragement, and confidence in their children's mathematics ability (for examples of items, see Appendix 1). The exploratory analysis showed that there was a statistically significant positive correlation between the student-level predictors (except for gender and age) with the indicators (see Appendix 2, Table A2.1). As expected, inter-correlations between the baseline indicators were relatively high.

Explanatory variables at the classroom level

On the student questionnaire, students were asked to indicate their opinion about the teaching of math in their classroom. The responses were aggregated at the classroom level; hence, we used class means. The reliability coefficients of the aggregated variables were calculated using Equation (1) given in Appendix 3. In particular, students were asked to express their perceptions about the classroom learning environment (CLEARN, 5 items, reliability λ = .81), classroom assessment (CLASSESS, 4 items, λ= .79), classroom modeling (CLMODEL, 5 items, λ = .74), and classroom questioning (CLQUEST, 5 items, λ = .82).

These classroom variables were derived from the student questionnaire of Kyriakides et al. (2014). CLEARN measures students' perceptions of the actual classroom psychosocial circumstances in a school. CLASSESS measures the students' perception of the extent to which the teacher

Table 4. Descriptive statistics of the variables used in the study.

Student-level variables	*M*	*SD*	Classroom-level variables	*M*	*SD*
SES	49.68	26.01	Classroom learning environment (CLEARN)	77.59	3.93
Gender	55% girls		Classroom assessment (CLASSESS)	61.32	5.72
Age	14.17	1.26	Classroom questioning (CLQUEST)	62.02	5.51
Prior Math achievement (PMA)	51.21	16.24	Classroom modeling (CLMODEL)	65.54	5.54
Parents' beliefs/attitudes (PABELF)	73.94	18.27	Math teacher beliefs/attitudes (MTBELF)	73.19	4.90
Prior Math self-confidence (PRCONF)	67.80	18.94	Peer influence (PEER)	52.97	5.26
Prior usefulness of Math (PRUSE)	75.19	18.14	Proportion of girls in the class (CLGIRLS)	54.39	18.49
Prior enjoyment of Math (PRENJOY)	68.49	19.88	Class mean of SES (CLSES)	49.77	10.06
			Class mean of PABELF (CLPABELF)	83.52	5.40
Dependent variables			Class mean of PMA (CLPMA)	50.13	3.77
Math self-confidence (CONF)	60.76	17.15	Class mean of PRCONF (CLPRCONF)	67.52	5.64
Usefulness of Math (USE)	76.46	18.58	Class mean of PRUSE (CLPRUSE)	74.42	6.99
Enjoyment of Math (ENJOY)	62.97	19.66	Class mean of PRENJOY (CLPRENJOY)	68.11	4.71

provides regular feedback on assessments. High scores indicate high levels of providing feedback to improve the teaching and learning of math. CLMODEL measures students' perception of how the teacher helps them to develop their own strategies in solving math problems through modeling and explanation. A high score means that students use their own strategies, whereas a lower score implies that students remain dependent upon the teacher to tell them what to do next. CLQUEST measures the students' perception of how the teacher uses questions to raise their level of participation and achievement. A high score indicates fostering of interaction between the teacher and his/her students. The teacher beliefs and attitudes scale (MTBELF, $\lambda = .88$) indicates the students' perceptions of their teachers' beliefs and attitudes toward them as learners of mathematics. The peer influence scale (PEER, $\lambda = .87$) shows the perceptions of the influence of their classmates and/or schoolmates (for examples of items, see Appendix 1). Higher scores of MTBELF and PEER indicate more positive perceptions. All these student-level factors were aggregated within classes. The exploratory analysis showed that most of the classroom predictors significantly correlated with the three indicators, except classroom learning environment and classroom assessment (see Appendix 2, Table A2.2).

Last, we calculated the mean of the student intake characteristics for each class and used them as indicators of the class composition (proportion of girls in the class, CLGIRLS, class mean of SES (CLSES), of PABELF (CLPABELF), of PMA (CLPMA), of PRCONF (CLPRCONF), of PRUSE (CLPRUSE), and of PRENJOY (CLPRENJOY).

Descriptive statistics of the student-level and classroom-level variables are shown in Table 4.

Missing data

Missing data are an inevitable problem in longitudinal research. Some background variables were measured only once. In addition, data could be missing because the students were absent on the day of the test or for some other reason. We observe that in general the percentages of missing data for the attitudinal indicators were lower at the first measurement point than at the second measurement point, as shown in Table 5.

In order to deal with the missing data problem, we used the full information maximum likelihood (FIML) estimation method. Under the missing at random (MAR) condition, FIML directly estimates parameters and their standard errors and outperforms ad hoc methods such as pairwise deletion and stepwise deletion (J.W. Graham, 2009). In our study, we indeed do not have indications that missingness is not at random.

Table 5. Percentage of missing values in the sample of 4,819 students.

Measure	Time 1	Time 2
Socioeconomic status (SES)	7.1%	NA
Prior Math achievement (PMA)	8.7%	NA
Parents' beliefs/attitudes (PABELF)	7.7%	NA
Math self-confidence (CONF)	5.3%	8.7%
Usefulness of Math (USE)	4.5%	7.5%
Enjoyment of Math (ENJOY)	3.8%	7.3%
Classroom learning environment (CLEARN)	NA	6.8%
Classroom assessment (CLASSESS)	NA	6.0%
Classroom questioning (CLQUEST)	NA	6.5%
Classroom modeling (CLMODEL)	NA	6.0%
Math teacher beliefs/attitudes (MTBELF)	NA	6.9%
Peer influence (PEER)	NA	6.3%

NA = Not applicable.

Data analyses

Model specification

The multivariate two-level model is conceptualized as a three-level model, in which the scores on the three indicators of ATM (Level 1) are clustered within students (Level 2), who are clustered within classes (Level 3) (Hauck & Street, 2004; Yang, Goldstein, Browne, & Woodhouse, 2002). Level 1 variation is not specified because this level is used only to define the multivariate structure (Goldstein, 2011). Because of model complexity, the school level was not included. Univariate analyses showed that the findings are similar with and without the school level. Hence, the subsequent analysis is based on the multivariate two-level model with student-level and class-level variance-covariance matrices $\mathbf{\Sigma}$ and $\mathbf{\Omega}$ (see Appendix 3).

The multivariate multilevel model parameters were estimated using SAS 9.3 (PROC MIXED) with ML method. We used this method after we had performed Little's Missing Completely at Random (MCAR) test (1988), and the result was significant (p = .025). We concluded that the data were not MCAR. However, there was no indication that the data were not MAR. To facilitate the interpretation and comparison of the magnitude of effects, all continuous predictors were standardized over all classes to z scores. We first specified a null model (Model 0) for the attitudinal indicators to estimate the variance and covariance situated at the student and classroom levels. Next, four multivariate multilevel models were specified, including the explanatory variables. In Model 1, we included the student intake characteristics. For each outcome variable, only the corresponding prior measurement was included as a predictor. Prior measurements of other indicators were not included to avoid over-fitting the model. Model 2 was fitted to estimate the effect of the *class processes* along with the student characteristics. Model 3 was specified to estimate the effect of the *class composition* variables together with the student characteristics. Finally, we included the student and all classroom characteristics in Model 4.

To compare the degree of model fit of nested models, we looked at the difference between the deviance (−2 Res Log Likelihood) of models, which follows a chi-square distribution with degrees of freedom equal to the difference in the number of parameters estimated.

Results

Table 6a shows the estimated variances and covariances at both levels in the null model. It also shows that the estimated covariances between each pair of indicators were significant and positive at both levels and were higher at the student level than at the class level. These observations indicate that, within and between classes, the higher the students score on one indicator, the higher they also score on the two other indicators.

Table 6a. Parameter estimates for the fixed and random effects.

	Null Model Model 0			Student intake characteristics Model 1		
Variable	CONF	USE	ENJOY	CONF	USE	ENJOY
Intercept	**60.40**(.57)	**75.95**(.82)	**62.51**(.75)	**60.67**(.56)	**74.54**(.84)	**62.93**(.77)
Student level						
SES				.14(.26)	.07(.28)	.37(.30)
GENDER				−.20(.53)	**2.80**(.58)	−.49(.61)
AGE				−**.77**(.27)	−.32(.29)	−.51(.31)
PMA				**1.87**(.27)	**.71**(.29)	**1.81**(.31)
PABELF				**.94**(.28)	**1.27**(.31)	**1.46**(.32)
PRCONF				**2.24**(.24)		
PRUSE					**1.60**(.28)	
PRENJOY						**1.96**(.26)
Within-class covariance matrix						
CONF	**273.3**(6.0)			**260.3**(5.8)		
USE	**109.2**(4.8)	**299.2**(6.6)		**102.7**(4.6)	**290.1**(6.4)	
ENJOY	**156.4**(5.4)	**146.5**(5.5)	**349.1**(7.7)	**142.5**(5.1)	**137.5**(5.3)	**332.3**(7.4)
Between-class covariance matrix						
CONF	**19.51**(3.9)			**12.76**(2.9)		
USE	**27.64**(5.2)	**45.92**(8.2)		**20.96**(4.2)	**40.25**(7.4)	
ENJOY	**23.81**(4.8)	**35.09**(6.8)	**36.37**(6.9)	**17.91**(3.7)	**29.94**(5.99)	**30.60**(5.97)
−2 log-likelihood	106932.1			105221.3		

Notes: Coefficients in bold are significant at $p < .05$; SES = socioeconomic status; PMA = prior mathematics achievement; PABELF = parents' beliefs & attitudes; PRCONF = prior mathematics self-confidence; PRUSE = prior usefulness of mathematics; PRENJOY = prior enjoyment of mathematics.

Based on the covariance matrix, we calculated correlations among the indicators and proportion of variance situated at each level (Table 7). Table 7 shows that the three indicators are highly correlated at the class level, where the correlations are at least 0.86. It also shows that the variances in the three indicators were mostly situated at the student level, with at least 86.7% of the total variance at the student level.

Table 6a, Model 1, shows that SES had no significant effect on any of the attitudinal indicators. The results showed that, compared to boys, girls found mathematics more useful. For mathematics self-confidence and enjoyment of mathematics, no significant gender effect was found. The results also showed that age had only a significant effect on mathematics self-confidence in favor of old students in the same class, and prior mathematics achievement and parental beliefs were positive and significant predictors across the attitudinal indicators (all $p < .001$). Similarly, for each outcome, the corresponding baseline indicator was a significant predictor ($p < .001$). Adding the intake characteristics had an impact on the covariances at the student level, and influenced the variances and covariances, which both decreased at the classroom level.

Among the class processes, classroom assessment consistently showed a significant negative association with the three attitudinal indicators (Table 6b, Model 2). The classroom questioning predicted greater mathematics self-confidence ($p = .02$) and enjoyment of mathematics ($p = .005$), and classroom modeling predicted greater endorsement of the three indicators. Mathematics teacher beliefs and attitudes ($p = .02$) had a negative influence on enjoyment of mathematics, and peer influence ($p = .02$) had a positive effect on enjoyment of mathematics. Among the class composition variables (Model 3), class mean of SES ($p = .004$) had a significant negative association with perceived usefulness of mathematics, the class mean of parental beliefs was found to be a significant predictor across the three attitudinal indicators ($p < .0001$), and class mean of prior enjoyment ($p = .02$) had a significant negative association with enjoyment of mathematics.

When the student and all class characteristics were introduced into the model (Model 4), classroom assessment ($p = .003$) had a significant negative association with mathematics self-confidence; classroom questioning ($p = .03$), teacher beliefs and attitudes ($p = .02$), peer influence ($p = .04$), and class mean of SES ($p = .003$) were negatively associated with usefulness of mathematics; classroom modeling ($p = .04$) and class mean of PRUSE ($p = .01$) were significantly predictive of greater endorsement of perceived usefulness of mathematics; and class mean of parents' beliefs

Table 6b. Parameter estimates for the fixed and random effects.

	Student intake characteristics + class process Model 2			Student intake characteristics + class composition Model 3			Student & all characteristics together Model 4		
Variable	CONF	USE	ENJOY	CONF	USE	ENJOY	CONF	USE	ENJOY
Intercept	**60.79**(.47)	**74.77**(.69)	**63.06**(.62)	**60.78**(.41)	**74.74**(.45)	**62.98**(.53)	**60.82**(.40)	**74.78**(.42)	**63.01**(.51)
Student level									
SES	0.15(.26)	0.06(.28)	0.41(.30)	0.06(.27)	−0.07(.28)	0.29(.30)	0.05(.27)	−0.07(.28)	0.29(.30)
GENDER	−0.17(.53)	**2.84**(.58)	−0.46(.61)	−0.10(.55)	**3.00**(.58)	−0.24(.62)	−0.15(.55)	**2.96**(.58)	−0.28(.62)
AGE	**−0.76**(.27)	−0.28(.29)	−0.54(.31)	**−0.70**(.27)	−0.20(.29)	−0.47(.31)	**−0.70**(.27)	−0.19(.28)	−0.48(.31)
PMA	**1.90**(.27)	**0.68**(.29)	**1.91**(.31)	**1.83**(.27)	**0.59**(.29)	**1.77**(.31)	**1.85**(.27)	**0.60**(.29)	**1.81**(.31)
PABELF	**0.94**(.28)	**1.22**(.31)	**1.54**(.32)	**0.80**(.28)	**1.04**(.30)	**1.35**(.31)	**0.86**(.28)	**1.10**(.30)	**1.46**(.32)
PRCONF	**2.26**(.24)			**2.24**(.24)			**2.23**(.24)		
PRUSE		**1.57**(.28)			**1.48**(.28)			**1.48**(.28)	
PRENJOY			**1.97**(.26)			**1.99**(.26)			**1.97**(.26)
Classroom level									
CLEARN	−0.28(.43)	−0.55(.70)	−1.10(.60)				−0.09(.33)	−0.04(.36)	−0.85(.45)
CLASSESS	**−1.27**(.41)	**−1.52**(.70)	**−1.22**(.59)				**−0.91**(.31)	−0.53(.33)	−0.53(.46)
CLQUEST	**1.21**(.52)	0.36(.86)	**2.06**(.73)				0.63(.41)	**−0.94**(.42)	**1.11**(.57)
CLMODEL	**1.66**(.49)	**3.68**(.80)	**2.15**(.69)				0.36(.41)	**0.90**(.43)	0.24(.57)
MTBELF	−0.67(.44)	−0.68(.70)	**−1.40**(.60)				−0.66(.40)	**−1.03**(.46)	**−1.05**(.52)
PEER	0.96(.51)	1.23(.81)	**1.59**(.70)				−0.09(.43)	**−.96**(.47)	0.42(.60)
CLSES				−0.58(.41)	**−1.34**(.47)	−0.28(.59)	−0.36(.42)	**−1.30**(.44)	−0.30(.58)
CLGIRLS				−0.16(.33)	−0.32(.37)	−0.66(.46)	0.12(.32)	−0.16(.35)	−0.30(.46)
CLPMA				−0.51(.38)	−0.74(.42)	−0.57(.55)	−0.26(.38)	−0.54(.40)	−0.01(.55)
CLPABELF				**4.01**(.40)	**7.15**(.45)	**5.59**(.57)	**3.32**(.47)	**7.02**(.50)	**4.63**(.67)
CLPRCONF				−0.17(.36)			0.40(.48)		
CLPRUSE					0.20(.40)			**1.28**(.52)	
CLPRENJOY						**−1.24**(.53)			−0.49(.63)
Within-class covariance matrix									
CONF	**260.1**(5.8)			**260.1**(5.8)			**260.0**(5.8)		
USE	**102.6**(4.6)	**290.2**(6.4)		**102.5**(4.6)	**289.9**(6.4)		**102.4**(4.6)	**290.0**(6.4)	
ENJOY	**142.3**(5.1)	**137.5**(5.3)	**332.2**(7.4)	**142.2**(5.1)	**137.2**(5.3)	**331.9**(7.3)	**142.1**(5.1)	**137.1**(5.3)	**331.8**(7.3)
Between-class covariance matrix									
CONF	**5.12**(1.6)			1.03(.92)			.27(.79)		
USE	**9.56**(2.3)	**22.04**(4.6)		−.06(.74)	**2.18**(1.12)		−.57(.60)	.37(.90)	
ENJOY	**6.87**(1.9)	**13.76**(3.3)	**13.60**(3.2)	1.04(1.05)	.04(1.13)	**5.72**(1.95)	.28(.86)	.27(.91)	**4.11**(1.64)
−2 log likelihood	105059.6			105157.6			105017.2		

Notes: Coefficients in bold are significant at $p < 0.05$; SE in parenthesis; SES = socioeconomic status; PMA = prior mathematics achievement; PABELF = parents' beliefs and attitudes; PRCONF = prior math self-confidence; PRUSE = prior usefulness in mathematics; PRENJOY = prior enjoyment in mathematics; CLEARN = classroom learning environment; CLASSESS = classroom assessment; CLQUEST = classroom questioning; CLMODEL = classroom modeling; MTBELF = mathematics teacher beliefs & attitudes; PEER = peer influence; CLSES = class mean of SES; CLGIRLS = proportion of girls in the class; CLPMA = class mean of PMA; CLPABELF = class mean of parents' beliefs & atitudes; CLPRCONF = class mean of PRCONF; CLPRUSE = class mean of PRUSE; CLPRENJOY = class mean of PRENJOY.

Table 7. Correlations of three attitudinal indicators at the student and classroom level.

	Correlations					
	Student level			Classroom level		
Indicator	1.	2.	3.	1.	2.	3.
1. Math self-confidence	1			1		
2. Usefulness of Math	0.38	1		0.92	1	
3. Enjoyment of Math	0.51	0.45	1	0.89	0.86	1

and attitudes ($p < .0001$) was a significant predictor of the three indicators. Enjoyment of mathematics was positively influenced by classroom questioning ($p = .05$), but negatively influenced by teacher beliefs and attitudes ($p = .04$).

We used the formula given by Tymms (2004) to calculate the effect sizes of the significant predictors. We found that the effect sizes of the student-level predictors were relatively small (ranging from 0.2 to 0.3), whereas those of the classroom-level predictors were relatively moderate or large (ranging from 0.4 to 1.8) ($\leq$ 0.2 is a small effect size, 0.50 is a moderate effect size, and $\geq$ 0.80 is a large effect size; Cohen, 1992).

Model fit

When the student intake characteristics were added, the model improved significantly ($\chi^2(18) = 1710.8, p < .0001$; compare Model 0 and 1). Entry of the class processes or the class-composition variables further improved the model, ($\chi^2(18) = 68.1, p = .0001$; compare Models 1 & 2), and ($\chi^2(15) = 161.7, p < .0001$; compare Model 1 & 3), respectively. When the student and all classroom characteristics were introduced into Model 4, there was a significant improvement in the model($\chi^2(51) = 1914.9, p < .0001$; compare Models 0 & 4), ($\chi^2(33) = 204.1, p < .0001$; compare Models 1 & 4), ($\chi^2(15) = 136, p < .0001$; compare Models 2 & 4), and ($\chi^2(18) = 42.4, p < .0001$; compare Models 3 & 4).

Table 8 shows that the student characteristics explained at least 3.1% and 12.2% of the variance in the three attitudinal indicators at the student and the classroom levels, respectively. Together with the student characteristics, the class processes and class composition explained at least 51.9% and 84.3% of the class variance, respectively, in the three indicators. The student characteristics and all classroom characteristics explained at least 3.1% and 88.7% of the variance in the three indicators at the student and classroom level, respectively.

Table 8. Percentage of variance of the three attitudinal indicators explained at two levels.

	Decomposition of total variance	Variance explained by student characteristics	Variance explained by student characteristics and class processes	Variance explained by student characteristics and class composition	Variance explained by student and classroom characteristics
Level	Model 0	Model 1	Model 2	Model 3	Model 4
Math self-confidence					
Student	93.3%	4.8%	4.8%	4.8%	4.9%
Classroom	6.7%	34.6%	73.7%	94.7%	98.6%
Usefulness of math					
Student	86.7%	3.1%	3.0%	3.1%	3.1%
Classroom	13.3%	12.2%	51.9%	95.2%	99.2%
Enjoyment of math					
Student	90.6%	4.8%	4.8%	4.9%	4.9%
Classroom	9.4%	15.9%	62.6%	84.3%	88.7%

Discussion

The multivariate multilevel analysis presented in our study was used to illustrate an approach to ATM in the presence of multiple indicators. The main finding of this study is that mathematics self-confidence, perceived usefulness, and enjoyment of mathematics are strongly and positively correlated, but have partly unique associations with student and classroom predictors.

Our findings support previous studies on the effect of student characteristics on ATM. A surprising result was that SES was not significantly associated with any of the indicators of ATM. This seems to suggest that students' ATM is irrespective of their family background. It is highly possible that Ugandan students from high-SES families as well as low-SES families can develop a positive ATM. Our results showing that girls had higher scores on the usefulness of mathematics compared to boys support the findings of Sanchez et al. (2004), though they contradict other studies (e.g., Kaiser-Messmer, 1993). In addition, that Ugandan female students see the value of mathematics can be enhanced by the 1.5 bonus points which are given every year to females since 1990 upon entry into all state universities with mathematics-based courses. In line with previous findings (Mohd et al., 2011), prior mathematics achievement positively predicts all three attitudinal indicators. This finding seems to suggest that high achievers develop a more positive attitude than lower achievers (Farooq & Shah, 2008; Yara, 2009), but consistent with Goodykoontz (2009), also attitudes and beliefs of parents have positive effects on their children's ATM. The results also indicate that baseline attitudinal indicators are important factors for predicting to what extent students have confidence in mathematics, how much they find mathematics useful and enjoy the subject.

The results of our study suggest that the teaching of mathematics is associated with students' ATM. The negative effect of classroom assessment on the three attitudinal indicators indicates that the more the teachers offer feedback on assessments, the less the students have confidence, interest, and enjoyment in mathematics. Probably this is in line with the observation of Van Landeghem, Van Damme, Opdenakker, De Fraine, and Onghena (2002), who observed that more feedback predicts a lower academic self-concept. Our results seem to suggest that students do not experience the feedback as useful, since they often consider only the marks and grades as important. Once mathematics tests or papers are returned, students often ignore teacher comments and constructive feedback on the papers. But is the exam-oriented education system of Uganda to blame? This is probably partly so, because the exam-based system distorts motivation and learning by overemphasizing the importance of grades as measures of students' abilities. As a result, some teachers give mostly negative feedback, when the achievement is poor. Classroom questioning which significantly boosts the students' self-confidence and enjoyment of mathematics may reflect greater engagement of teachers with their students. In addition, the more the teachers improve their classroom modeling, the more the students perceive mathematics as useful for their present and future life, the more they become confident in solving mathematics-related problems and increase their interest in the subject. This is in line with Higgins (1997), who observed that students who received modern problem-solving instruction showed more positive attitudes toward the usefulness of mathematics.

Generally, the class mean of parental beliefs and attitudes is positively associated with the three indicators. This suggests that parents not only influence their children's mathematics attitudes but probably also what is happening in their children's class (see also Opdenakker & Van Damme, 2004). Indeed, parents have a great role to play in forming their children' attitudes.

As described above, some teaching-related factors were predictive of particular indicators of ATM. Moreover, the variance at the student and classroom level differed in relative magnitude. Both the class processes and composition are important in explaining the variance in each of the attitudinal indicators. Nevertheless, the class composition is more important than class processes in explaining the variance in these indicators at the classroom level. What remained consistent across ATM outcomes was the effect of parental beliefs at the student-level and the class-level aggregate.

Our results are supportive of maintaining multidimensional distinctions of ATM with respect to possible interventions aimed at teaching-related factors. Hence, educational effectiveness research should focus on multiple outcomes in considering effectiveness. And overall efforts to improve ATM may necessitate interventions aimed at parents and students, and may thus alter the school climate with greater parental investment in mathematics training.

Strengths, limitations, and future research

Longitudinal data enable us to study dynamic relationships between variables and to model the differences among students over time. Snijders and Bosker (2012), and Tabachnick and Fidell (2007) mention some strengths of using a multivariate multilevel approach over separate univariate analyses. These include a gain in statistical power which reduces the risk of both Type I and Type II statistical errors, and a greater insight into correlations between outcome variables, and testing the joint effect as well as the differences in the effect of a significant predictor on different response variables.

Our study has some limitations. Our results point to associations among variables. Hence, no conclusions can be drawn on the causal effects. Our study is limited to student- and classroom-level variables. School-level variables could be explored in future research to assess their impact on the indicators of ATM. Our study relied on student ratings in constructing teaching characteristics to measure the quality of mathematics teaching. Although these students' perceptions are very informative of the classroom environment, also qualitative data collection strategies (classroom observations and teacher reports through interviews) could be used in future research.

Another limitation was the questionable reliability of the scales since they had low Cronbach's alpha coefficients. The alpha values of the self-confidence scale were just on the cut-off point of 0.70 for internal consistency as proposed by Majeed, Darmawan, and Lynch (2013), whereas the alpha values of the scales of perceived usefulness and enjoyment of mathematics were below this cut-off point. The items of these scales were retained on the grounds that for research purposes a lower internal consistency can still be regarded as acceptable (e.g., Suhr & Shay, 2009, suggested a cut-off of 0.60). They were mainly retained because their mean inter-item correlation (ρ; Equation [8] in Appendix 3), was still within the acceptable range of 0.15 to 0.20 as recommended by Clark and Watson (1995). For instance, ρ of PRENJOY was .18.

Despite these limitations, our study raised some important implications for educational practice and policy.

Implications

Educators, especially mathematics teachers, need to take note of the close link of the baseline mathematics achievement and parental beliefs with mathematics self-confidence, usefulness, and enjoyment of mathematics as they try to help students to cultivate a positive mathematics attitude. Policymakers should engage parents in the process of fostering and forming students' positive ATM. Probably, this can be achieved by promoting Parent–Teacher Associations in Ugandan schools. It is also imperative for parents themselves to develop a positive mathematics attitude so that they can in turn help their children develop a positive attitude.

Assessment is seen as an integral part of teaching (Creemers & Kyriakides, 2008), and formative assessment in particular has been shown to be one of the most important factors associated with effectiveness at all levels, especially at the classroom level (e.g., Shepard, 1989). Information gathered from assessments should be used by teachers to diagnose their students' academic needs, as well as to evaluate their own practice (Black & Wiliam, 1998; Marshall & Drummond, 2006). Therefore, mathematics teachers are expected to provide constructive feedback on assessment so as to monitor and improve teaching and students' confidence, interest, and enjoyment in learning mathematics. Parents should support their children's learning by receiving feedback on their children's academic progress from teachers. Students should pay attention to teachers' remarks or comments, rather than emphasize marks or grades.

Classroom questioning is an important factor for students' self-confidence and enjoyment of mathematics. This implies that a mathematics teacher should ask questions in view of actively engaging students in the learning process. Additionally, the teacher should encourage students to ask questions, to which the teacher should offer a response. Hence, students will develop confidence and be interested in what they learn. However, the current questioning does not help students to value mathematics in their daily and later life. Therefore, teachers should ask questions geared to everyday experiences.

Classroom modeling is significantly linked with the three aspects of ATM. Hence, teachers should assist students to utilize proven strategies and/or develop their own strategies to help them develop mastery and confidence in solving math-related problems (Kyriakides et al., 2014), see the importance of mathematics in their everyday present life and future careers, and cultivate a deeper interest in the subject.

Surprisingly, teachers' beliefs and attitudes are negatively associated with students' usefulness and enjoyment of mathematics. Teachers also need to realize that how they teach, how they feel about the subject matter, and how they behave in and out of the classroom affects students' attitudes.

Peer influence has a positive effect on enjoyment of mathematics, but a negative effect on the usefulness of the subject. Here, we have indications that in classes with a high emphasis on enjoyment, students will encourage each other to like the subject. However, if the majority of students do not value mathematics, they will negatively influence the general disposition of the whole class.

In conclusion, we recommend that teachers should develop positive relationships with students and explore various teaching techniques and classroom activities, involving active teaching-learning processes. Because education plays an important role in the economic growth and development of a country (Hanushek & Woessmann, 2008), policymakers and educational practitioners must devise strategies to create and inculcate positive attitudes in students for them to perform well in studies. This is in view of steering developing countries such as Uganda to advance economically and technologically. The educational stakeholders should organize seminars and workshops (ongoing in-service training) for parents, teachers, and school heads to enhance and promote students' positive ATM. Our findings confirm that prior mathematics achievement is significantly linked with each of the three attitudinal indicators. Hence, future research can adopt a structural equation modeling or path analytic approach to explore the directionality of the relationship between each of the attitudinal indicators and mathematics achievement across time.

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Appendix 1. Scales and items

See https://ppw.kuleuven.be/o_en_o/HKlink

Appendix 2. Descriptives of variables at student and class level

See https://ppw.kuleuven.be/o_en_o/HKlink

Appendix 3. Statistical issues and calculations

Snijders and Bosker's (2012) formula for reliability coefficient is

$$\lambda_j = \frac{\tau^2}{\tau^2 + \frac{\sigma^2}{n_j}} \tag{1}$$

where τ^2 = variance between classes, σ^2 = variance within classes (between students), and n_j = number of students within class j. Since not all classes are of the same size, the mean class size was entered for n_j to get an idea of the reliability of the measure in a classof average size (Lüdtke et al., 2008).

The multivariate multilevel model is given as:

$$Y_{mij} = \sum_{t=1}^{3} \beta_{0t} z_{mt} + \sum_{t=1}^{3} \beta_{1t} z_{mt} X_{mij} + \sum_{t=1}^{3} \beta_{2t} z_{mt} W_{mj} + \sum_{t=1}^{3} u_{0tj} z_{mt} + \sum_{t=1}^{3} \varepsilon_{0tij} z_{mt} \tag{2}$$

where

$$z_{mt} = \begin{cases} 1, if \ t = m \\ 0, otherwise \end{cases}$$

Y_{mij} is the value of the m-th outcome of the i-th student of the j-th class, with m = 1, 2, 3 (1 = math self-confidence, 2 = perceived usefulness of math, 3 = usefulness of math), i = 1, ……., n_j, j = 1,……, J = 78 classes, and t indexes the set of indicators. X_{mij} is the vector of student-level covariates; W_{mj} is the vector of classroom-level covariates; ε_{0tij} are student-level errors and are assumed independent across students, and u_{0tj} are classroom-level errors and are assumed independent across classes. Since the school level is omitted, we assume that u_{0tj} also accounts for school unobserved factors.

The student level errors ε_{0tij} are assumed to be multivariate normal with zero means and covariance matrix

$$\sum = \begin{bmatrix} \varepsilon_{1ij} \\ \varepsilon_{2ij} \\ \varepsilon_{3ij} \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{e1} & & \\ \sigma_{e12} & \sigma^2_{e2} & \\ \sigma_{e13} & \sigma_{e23} & \sigma^2_{e3} \end{bmatrix} \right) \tag{3}$$

whereas class-level errors u_{0tj} are assumed to be multivariate normal with zero means and covariance matrix

$$\Omega = \begin{bmatrix} u_{1ij} \\ u_{2ij} \\ u_{3ij} \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau^2_{u1} & & \\ \tau_{u12} & \tau^2_{u2} & \\ \tau_{u13} & \tau_{u23} & \tau^2_{u3} \end{bmatrix} \right) \tag{4}$$

The matrices in (3) and (4) include covariances beside variances. This implies that the indicators of ATM can be correlated at both student and class levels. The covariance for the m-th and m'-*th* outcome over students within classes j is given by:

$$\text{cov}(\varepsilon_{0mij}, \varepsilon_{0m'ij}) = \sigma_{\varepsilon,mm'} \tag{5}$$

These estimates of covariance can be used to calculate the degree of correlation $r_{mm'}$ between indicator m and m' at the student level:

$$r_{mm'} = \frac{\sigma_{e,mm'}}{\sqrt{\sigma^2_{e,m} \times \sigma^2_{e,m'}}} \tag{6}$$

The intra-class correlation coefficient at the student and class level is estimated, respectively, as:

$$ICC_{MVML} = \sigma^2_{em}\left(\sigma^2_{em} + \tau^2_{um}\right)^{-1} \quad 0 < ICCMVML < 1 \tag{7}$$

The mean inter-item correlation is

$$\rho = \frac{\alpha}{n - (n-1)\alpha} \tag{8}$$

where ρ = an estimator of reliability, α = coefficient alpha, and n = the number of items in the scale.